1. $$g(x) = (2 + ax)^8 \qquad \text{where } a \text{ is a constant}$$

Given that one of the terms in the binomial expansion of g(x) is $3402x^5$

(a) find the value of a.

(4)

Using this value of a,

(b) find the constant term in the expansion of

$$\left(1 + \frac{1}{x^4}\right)(2 + ax)^8$$

(3)

a)

By the binomial expansion, the x^5 term is :

BINOMIAL SERIES :

$$\binom{8}{5} \times 2^{8-5} (ax)^5$$

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} \times b + \dots + b^n$$

$$= \frac{8!}{5!(8-5)!} \times 2^3 \times a^5 x^5 \quad (1)$$

where : $\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$

(1)

$$= 448a^5x^5 = 3402x^5$$

$$= 448a^5 = 3402$$

$$= \quad a^5 = \frac{3402}{448} = \frac{243}{32} \quad (1)$$

$$a = \frac{\sqrt[5]{243}}{\sqrt[5]{32}}$$

$$a = \frac{3}{2} \quad (1)$$

b)

The first constant is 2^8 (1) (the constant of the expansion $\times 1$) = 256.

The second constant is the x^4 term of the expansion $\times \frac{1}{x^4}$ term.

$= {}^8C_4 \times 2^4 a^4 = 70 \times 16 \times \frac{181}{16} = 5670$

(1)

The constant term is $256 + 5670 = 5926$ (1)

2. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(3 - \frac{2x}{9}\right)^8$$

giving each term in simplest form.

(4)

$$f(x) = \left(\frac{x-1}{2x}\right)\left(3 - \frac{2x}{9}\right)^8$$

(b) Find the coefficient of x^2 in the series expansion of $f(x)$, giving your answer as a simplified fraction.

(2)

1st term　　2nd term

a) $\left(3-\frac{2x}{9}\right)^8 = \binom{8}{0}(3)^8\left(-\frac{2x}{9}\right)^0 + \binom{8}{1}(3)^7\left(-\frac{2x}{9}\right)^1 +$ 1 1

3rd term　　4th term

$\binom{8}{2}(3)^6\left(-\frac{2x}{9}\right)^2 + \binom{8}{3}(3)^5\left(-\frac{2x}{9}\right)^3 + \dots$ 1

$= 6561 - 3888x + 1008x^2 - \frac{488}{3}x^3 + \dots$ 1

b) $\left(\frac{x-1}{2x}\right)\left(3-\frac{2x}{9}\right)^8$

$\left(\frac{x}{2x} - \frac{1}{2x}\right)\left(3-\frac{2x}{9}\right)^8$

$\left(\frac{1}{2} - \frac{1}{2x}\right)\left(6561 - 3888x + 1008x^2 - \frac{488}{3}x^3 + \dots\right)$

coefficient of x^2: $\left(\frac{1}{2} \times 1008\right) + \left(-\frac{1}{2} \times -\frac{488}{3}\right)$ 1

$= 504 + \frac{244}{3}$

$= \frac{1756}{3}$ 1

3. Find, in simplest form, the coefficient of x^5 in the expansion of

$$\left(5 + 8x^2\right)\left(3 - \frac{1}{2}x\right)^6$$

(5)

Since the first term is $(5 + 8x^2)$, we need term of x^5 and x^3 from the second term to find the coefficient of x^5 from the expansion.

$: \; 5 \times x^5$ term

$: \; 8x^2 \times x^3$ term

x^5 term : ${}^6C_5 \times 3^1 \left(-\frac{1}{2}x\right)^5 = -\frac{9}{16}x^5$ (1)

x^3 term : ${}^6C_3 \times 3^3 \left(-\frac{1}{2}x\right)^3 = -\frac{135}{2}x^3$ (1)

coefficient of x^5 in the expansion :

$\left(5 \times -\frac{9}{16}x^5\right) + \left(8x^2 \times -\frac{135}{2}x^3\right)$ (1) (1)

$= -\frac{45}{16}x^5 + (-540x^5)$

$= \left(-\frac{45}{16} - 540\right)x^5$

$= \frac{-8685}{16}$ (1)